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Two Stages One Qubit Quantum Teleportation via An Arbitrary Entangled Two Qubits Quantum Channel

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ABSTRACT

In this work, we consider two stages one qubit quantum teleportation protocol via an arbitrary entangled two qubits state. In the first stage, Alice use XOR gate and computational basis measurement, while for the second stage, Alice use Hadamard gate and another computational basis measurement. Alice sent measurement result to Bob via one classical bit each. As the result, we show that this teleportation can has unit fidelity if the channel used has non-zero two terms and maximally entangled. Then the protocol is applicable if the channel is one of four Bell's state.

Keywords: *two stages, one qubit, quantum teleportation, two qubits channel*

1. INTRODUCTION

Quantum teleportation is a unique quantum mechanics phenomenon with multifarious applications, from secure quantum communication [1], distributed quantum computer [2], quantum repeater [3], quantum cryptography [4] to quantum internet [5]. Firstly introduced by Bennet [6], the teleportation allows Alice, as the sender, to send an arbitrary qubit to Bob, as the receiver, via a quantum channel assisted with only two-bit classical channel. The classical channel was used to help Bob to unitarily reconstruct the initial qubit information with certainty. Thus, theoretically, the teleportation can send infinity type of information with only two classical bits.

Over the past years, different types of teleportation protocols were proposed. Ting proposed Controlled Quantum Teleportation protocol that includes additional character Charlie, who act as a controller [7], the special feature of this scheme is that teleportation success depends on the agreement of the controller to co-operate. Another alternative scheme was Bidirectional Quantum Teleportation protocol [8], that allowing Alice and Bob act as the sender and the receiver at the same time, various scheme and media in this protocol had already developed [9-14]. And there still other types of protocol that had been proposed.

In all the protocols mentioned above, the measurement is performed using an entangled state as the bases. Lo, et al. introduced an alternative way, i.e., for one qubit standard case, by introducing two stages protocol [15]. They use one of bell states [1] as the quantum channel and perform two measurements in computational bases. However, in a real situation, the entangled two qubits state used in teleportation may vary. Then, a complete analysis of the quantum channel state should be done.

In this work, we proposed two stages quantum teleportation protocol via an arbitrary channel state. We do not change the steps of the former protocol. Hence, the analysis will be done for the channel used only. We will show what kind of state can be used in this type of protocol, i.e., with unit fidelity. Furthermore, we will show the unitary operation that should be used to reconstruct the initial qubit state.

This article is composed of four sections. The first section is the introduction. The second section describes two stages quantum teleportation protocol via an arbitrary entangled two qubits state. The third section discusses all of the possible results and the implications. Finally, the fourth section is the conclusion.

2. TWO STAGES QUANTUM TELEPORTATION PROTOCOL

In this section, we consider two stages one qubit quantum teleportation via an arbitrary entangled two qubits state. Suppose Alice, as a sender, has qubit of the form

$$|\varphi\rangle_a = x_0|0\rangle + x_1|1\rangle, \tag{1}$$

with x_0 and x_1 are complex number satisfying normalization condition $|x_0|^2 + |x_1|^2 = 1$. Bob, as the receiver, share an entangled two qubit state with Alice, as

$$|\psi\rangle_{AB} = c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle, \tag{2}$$

with c_i for $i = \{1,2,3,4\}$ are complex numbers satisfying normalization condition $\sum_{j=1}^{4}|c_j|^2 = 1$. Index A and B indicate that qubit A and B belong to Alice and Bob, respectively.

Alice mixes the states as $|\Psi_{mix}\rangle_{aAB} = |\varphi\rangle_a \otimes |\psi\rangle_{AB}$, and can be written as

$$\begin{aligned}|\Psi_{mix}\rangle_{aAB} = {} & x_0c_1|000\rangle + x_1c_1|100\rangle \\ & +x_0c_2|001\rangle + x_1c_2|101\rangle \\ & +x_0c_3|010\rangle + x_1c_3|110\rangle \\ & +x_0c_4|011\rangle + x_1c_4|111\rangle.\end{aligned} \tag{3}$$

The teleportation is divided into two stages and presented in the following two sections. Teleportation scheme is shown in Figure 1.

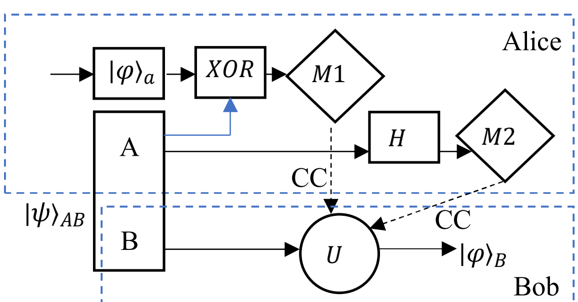

Figure 1 Two stage quantum teleportation scheme. XOR, M1, H, M2, U, and CC refers to XOR gate, Alice's First Stage Measurement, Hadamard Gate, Alice's Second Stage Measurement, Unitary Operation, and Classical Communication, respectively. The blue arrow show that the qubit become the control qubit on XOR Gate.

2.1. First Stage

In the first stage, Alice preforms XOR gate on her two qubits, i.e., qubit a and A, with qubit a as the target. Then, Eq. (3) become

$$\begin{aligned}|\Psi'_{mix}\rangle_{aAB} = {} & x_0c_1|000\rangle + x_1c_1|100\rangle \\ & +x_0c_2|001\rangle + x_1c_2|101\rangle \\ & +x_0c_3|110\rangle + x_1c_3|010\rangle \\ & +x_0c_4|111\rangle + x_1c_4|011\rangle.\end{aligned} \tag{4}$$

Alice performs computational basis projection measurement in qubit a, resulting two possible states. If the measurement result is $|0\rangle$, then Eq. (4) collapse to

$$\begin{aligned}|\Phi_{mix,0}\rangle_{AB} = {} & x_0c_1|00\rangle + x_0c_2|01\rangle \\ & +x_1c_3|10\rangle + x_1c_4|11\rangle.\end{aligned} \tag{5}$$

But, if the measurement result is $|1\rangle$, then Eq. (4) collapse to

$$\begin{aligned}|\Phi_{mix,1}\rangle_{AB} = {} & x_1c_1|00\rangle + x_1c_2|01\rangle \\ & +x_0c_3|10\rangle + x_0c_4|11\rangle.\end{aligned} \tag{6}$$

Alice sends the measurement result via one classical bit to Bob. This is the end of the first stage.

2.2. Second Stage

In the second stage, Alice perform Hadamard gate on qubit A, then the states in Eq. (5)-(6) is transformed into

$$\begin{aligned}|\Phi'_{mix,0}\rangle_{AB} = {} & (x_0c_1 + x_1c_3)|00\rangle + (x_0c_2 + x_1c_4)|01\rangle \\ & + (x_0c_1 - x_1c_3)|10\rangle + (x_0c_2 - x_1c_4)|11\rangle\end{aligned} \tag{7}$$

$$\begin{aligned}|\Phi'_{mix,1}\rangle_{AB} = {} & (x_1c_1 + x_0c_3)|00\rangle + (x_1c_2 + x_0c_4)|01\rangle \\ & + (x_1c_1 - x_0c_3)|10\rangle + (x_1c_2 - x_0c_4)|11\rangle.\end{aligned} \tag{8}$$

Alice performs computational bases projection measurement on qubit A. Then, for Eq. (7), if the measurement result is $|0\rangle$, or $|1\rangle$, the state collapse to

$$|\varphi_{0,0}\rangle_B = (x_0c_1 + x_1c_3)|0\rangle + (x_0c_2 + x_1c_4)|1\rangle \tag{9}$$
$$|\varphi_{0,1}\rangle_B = (x_0c_1 - x_1c_3)|0\rangle + (x_0c_2 - x_1c_4)|1\rangle, \tag{10}$$

respectively. Similarly, for Eq. (8),

$$|\varphi_{1,0}\rangle_B = (x_1c_1 + x_0c_3)|0\rangle + (x_1c_2 + x_0c_4)|1\rangle \tag{11}$$
$$|\varphi_{1,1}\rangle_B = (x_1c_1 - x_0c_3)|0\rangle + (x_1c_2 - x_0c_4)|1\rangle. \tag{12}$$

Alice sends the measurement result to Bob via another classical bit. For unit fidelity case, i.e., $\left|\langle\varphi|_a \otimes |\varphi_{m,n}\rangle_B\right|^2 = 1$ for $\{m,n\} = \{0,1\}$. Then there

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will be four sets of allowable constants as described in following section.

3. DISCUSSION

We consider unit fidelity case. Hence, not all the constant's value can be used. If we analyse the form of Eq. (9)-(12), there will be four possible sets, i.e., $c_2 = c_3 = 0$ and $c_1 = \pm c_4$; or $c_1 = c_4 = 0$ and $c_2 = \pm c_3$.

For $c_2 = c_3 = 0$ and $c_1 = c_4$, the channel used reduce to Bell State, i.e., $|\Phi\rangle^+ = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ [1]. And Bob's state become

$$|\varphi_{0,0}\rangle_B = x_0|0\rangle + x_1|1\rangle \tag{13}$$
$$|\varphi_{0,1}\rangle_B = x_0|0\rangle - x_1|1\rangle \tag{14}$$
$$|\varphi_{1,0}\rangle_B = x_1|0\rangle + x_0|1\rangle \tag{15}$$
$$|\varphi_{1,1}\rangle_B = x_1|0\rangle - x_0|1\rangle. \tag{16}$$

In this analysis, we exclude the global phase parameters. Then, Bob can perform appropriate unitary transformation in the form of $I, \sigma_z, \sigma_x, \sigma_x\sigma_z$ if Bob receives classical bit of the form 00, 01, 10, 11, respectively. For other cases, the analysis is done similarly. The results are shown in Table 1.

Table 1. List of cases, classical bit received by Bob, Bob's qubit state and unitary operator.

Cases	Classical Bit	Bob's qubit	Unitary Operator
$c_2 = c_3 = 0$ and $c_1 = c_4$	00	$x_0\|0\rangle + x_1\|1\rangle$	I
	01	$x_0\|0\rangle - x_1\|1\rangle$	σ_z
	10	$x_1\|0\rangle + x_0\|1\rangle$	σ_x
	11	$x_1\|0\rangle - x_0\|1\rangle$	$\sigma_x\sigma_z$
$c_2 = c_3 = 0$ and $c_1 = c_4$	00	$x_0\|0\rangle - x_1\|1\rangle$	σ_z
	01	$x_0\|0\rangle + x_1\|1\rangle$	I
	10	$x_1\|0\rangle - x_0\|1\rangle$	$\sigma_x\sigma_z$
	11	$x_1\|0\rangle + x_0\|1\rangle$	σ_x
$c_2 = c_3 = 0$ and $c_1 = c_4$	00	$x_1\|0\rangle + x_0\|1\rangle$	σ_x
	01	$x_1\|0\rangle - x_0\|1\rangle$	$\sigma_x\sigma_z$
	10	$x_0\|0\rangle + x_1\|1\rangle$	I
	11	$x_0\|0\rangle - x_1\|1\rangle$	σ_z
$c_2 = c_3 = 0$ and $c_1 = c_4$	00	$x_1\|0\rangle - x_0\|1\rangle$	$\sigma_x\sigma_z$
	01	$x_1\|0\rangle + x_0\|1\rangle$	σ_x
	10	$x_0\|0\rangle - x_1\|1\rangle$	σ_z
	11	$x_0\|0\rangle + x_1\|1\rangle$	I

The table shows that this protocol can only be used if the channel is in maximal entanglement. Hence, the channel should be one of Bell's States.

4. CONCLUSION

In this work, we analyze two stages one qubit quantum teleportation via an arbitrary entangled two qubits state. The calculation shows that the teleportation is successful, i.e., with unit fidelity, if the channel has two non-zero terms with maximal entanglement. Then, the channel should be one of four Bell's states. Hence, if the channel used is in non-maximal entanglement, this protocol cannot teleport with unit fidelity.

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