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LEARNING DESIGN FOR INTRODUCING *e*-GEOMETRICALLY

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Abstract: Introducing the Euler's number, or symbolized as e, is often done formally by teachers of the course of differential calculus at the college level. This makes students not really familiar with this number. In fact, the concept of this number is very often used or involved in mathematics and mathematics education study programs. For this reason, this study aims to provide alternative learning to introduce the Euler's number to students using a geometric approach. This type of research is design research using ADDIE model, which has reached the design stage. The results of this study are the result of the analysis stage, which consisted of learner analysis, instructional analysis, and instructional objective, and the results of the design stage, which emerge the learning phase to guide the students to construct the definition of Euler's number geometrically which consisted of six steps, namely: 1) sketching the exponential function; 2) sketching the logarithmic function; 3) answering the problems given regarding the function of the exponent and tangent to the function with the help of illustrations provided through the GeoGebra application; 4)through the steps to find the number e given, students are asked to explain the reasons that underlie the action of each step; 5) determining the value of the number e through the illustration in the GeoGebra application; and 6) constructing the definition the Euler's number geometrically.

Keywords: *Euler's Numbe; Learning Design; ADDIE Model*

A. INTRODUCTION

Differential calculus is one of the fundamental courses given in several study programs, such as mathematics study programs, mathematics education, or various engineering study programs. Especially in mathematics and mathematics education programs in Indonesia, almost all universities use the agreed curriculum under the Indonesian Mathematical Society (Indo-MS), which is a differential calculus course given in the first year. This course supports many further courses such as integral calculus, real analysis, multivariable calculus, differential equations, and others.

In fact, many students find it difficult with differential calculus course (Robert and Speer 2001). It is an area of mathematics perceived as the main source of failure at the undergraduate level because of its nature, which involves abstract and complex ideas and the way it is being taught to the students (Sahin, Cavlazoglu, and Zeytuncu 2015). Mathematics educators in various countries have carried out many learning designs because of the importance of this course so that students can master the concepts well (Jaafar and Lin 2017). Some of them use ICT integration as an approach to make their students understand the concept in calculus differential (Jaafar and Lin 2017) (Mendezabal and Tindowen 2018) (Montoya and Prada 2019), some others use certain learning models (Istiandaru et al. 2019).

One of the important concepts introduced in this course is related to *e*. In the real number system, the Euler's number is only introduced with its name and value, usually added by its discoverer, Leonhard Euler. In the discussion of function graphs, Euler's number usually appears in the discussion of exponential functions. In the discussion of limits, the Euler's number appears as the result of $\lim_{n\to 0}(1+n)^{1/n}$ atau $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$. This introduction is too abstract and demands that students should be forced to accept that the value of e is 2.7182…. In the discussion of derivatives, usually the derivative of $y = e^x$ is a special form of $y = a^x$ which is approximated by natural logarithms. In essence, the e always appears as something abstract, never given to students a concrete representation that represents it.

B. METHODS

This research is a design research type using ADDIE model, with stages are Analysis, Design, Develop, Implementation, and Evaluation (Aldoobie 2015); however, this article just still reaches the second stage, design. At the stage (analysis), the researcher conducted an analysis of the calculus books at the college level, which were commonly used for differential calculus lectures. In addition, unstructured interviews were conducted on students who had taken the classical differential calculus course to obtain data on students' understanding of the Euler's number. After that, the development of learning activities in the design stage was carried out to introduce the Euler's number to students geometrically.

C. RESULT & DISCUSSION

The results of this study are divided into two parts, namely the results of the analysis stage and the results of the design stage. The two results are presented below.

1. Analysis Stage Result

At this stage, several analyses were carried out, including 1) analysis of students; 2) instructional analysis; 3) preparation of learning objectives, and 4) analysis of learning objectives. Student analysis is needed to identify the needs of students, their level of understanding, what are the prerequisites for the material to be delivered, to the characteristics of students. Unstructured interviews were conducted with students who had taken differential calculus courses to get a clear picture of students' understanding of Euler's number. The results of the interview showed that students could not provide an explanation except for the value of e = 2.718… and the inventor was Leonhard Euler. This shows that not enough students understand this Euler's number.

The next step is the preparation of learning objectives. The ability that is expected to be achieved by students through structured learning designs is that students can construct a geometric definition of the Euler's number through guided discovery. These objectives can be achieved by requiring students to be able to: 1) explain the properties of the function, especially the exponential and logarithmic functions; 2) drawing graphs of functions, especially graphs of functions of exponents and logarithms; 3) explain the concept of tangent gradient; 4) using the GeoGebra application; 5) explain the concept of limit and its properties; and 6) describes the definition of a derivative.

2. Design Stage Result

The design stage is the next step in the ADDIE model. In this design phase, it focuses more on the preparation of learning strategies, which are manifested in the steps of guided discovery carried out on students so that students can achieve predetermined learning objectives, namely students can construct the definition of the Euler's number geometrically. Step by step of students constructs the definition of e geometrically as presented in Table 1.

Table 1. The Steps of students construct the definition of e geometrically

Steps	Assignment instructions to students and the expected response
Step 1: Sketching the exponential function	Sketch the exponential function $f(x) = a^x$ with various bases. For instances: $f(x) = 2^x, f(x) = 3^x$, dan $f(x) = 10^x$. Expected response: Students draw a graph of the exponential function according to the instructions

Steps	Assignment instructions to students and the expected response
Step 2: Sketching the logarithmic function	Sketch the logarithmic function $g(x) = \log_a x$ (which is the inverse of the exponential function) with the same base as step (1), such as $g(x) = \log_2 x$, $g(x) = \log_3 x$, dan $g(x) = \log_4 x$. Expected response: Students draw a graph of the logarithmic function according to the instructions
Step 3: answer the problems given regarding the function of the exponent and tangent to the function with the help of illustrations provided through the GeoGebra application.	What is the **basis** for the **exponential** function that has the slope of the tangent at point (0,1) of 1 (with the angle of slope 45°)? This problem is equivalent to what is the **basis** of the **logarithmic** function, which has the slope of the tangent at point (1,0) of 1? Use illustrations using GeoGebra via the following link https://www.geogebra.org/classic/qhrktmsx Expected response: Students guess the answer, for example: between 2.7 and 2.8
Step 4: through the steps to find the number e given, students are asked to explain the reasons that underlie the action of each step.	*Actually, someone named Leonhard Euler has found that number. Because of these findings, the symbol is symbolized e. "* *What is the real value of e? Take a look at the following discovery steps.* Let $g(x) = \log_e x$. Because of $g(x)$ has had the slope of the tangent 1 at point (1,0), then: $1 = g'(1)$ (1) $1 = \lim_{h\to 0} \frac{g(1+h)-g(1)}{h}$ (2) $1 = \lim_{h\to 0} \frac{\log_e(1+h)-\log_e 1}{h}$ (3) $1 = \lim_{h\to 0} \frac{1}{h}\log_e(1+h)$ (4) $1 = \lim_{h\to 0} \log_e(1+h)^{\frac{1}{h}}$ (5) $1 = \log_e\left(\lim_{h\to 0}(1+h)^{\frac{1}{h}}\right)$ (6) If the above equation is converted into exponential form, then $$e = \lim_{h\to 0}(1+h)^{\frac{1}{h}}$$ If the variable h is replaced by $1/h$, the form is obtained $$e = \lim_{h\to\infty}\left(1+\frac{1}{h}\right)^h$$ Expected response: (1) Because the function g (x) has a tangent slope of 1 (2) Based on the definition of a derivative (3) Based on the function formula g (4) Simplification of the form (3) by applying logarithmic properties (5) Change the form (4) by applying the logarithmic property (6) because the logarithmic function is continuous so $\lim_{x\to c} f(x) = f(\lim_{x\to c} x)$
Step 5: determine the value of the number e through	Use GeoGebra to find the value for e by dragging the h value over the following GeoGebra https://www.geogebra.org/graphing/vnghc6v7

Steps	Assignment instructions to students and the expected response
the illustration in the GeoGebra application	Expected response: students determine the value of *e*
Step 6: construct the definition of the Euler's number geometrically	Arrange the definition of the Euler's number geometrically by involving the exponential function and its slope. Expected response: Geometrically, the number e is the basis of the exponential function, which has the slope of the tangent at point (0,1) of 1 (with the angle of slope 45^0)

D. CONCLUSION

Euler's number in calculus differential course often introduced abstractly, without given intuitively. Some students only know this number by its value, symbol, and its inventor. This research results in learner analysis, instructional analysis, and learning objective formulation, as well as the instructional strategy of introducing Euler's number to students using the geometry approach. Six steps proposed to engage the students in constructing the definition of Euler's number geometrically are: 1) sketching the exponential function; 2) sketching the logarithmic function; 3) answering the problems given regarding the function of the exponent and tangent to the function with the help of illustrations provided through the GeoGebra application; 4) through the steps to find the number e given, students are asked to explain the reasons that underlie the action of each step; 5) determining the value of the number e through the illustration in the GeoGebra application, and 6) constructing the definition the Euler's number geometrically.

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