Stability Of a Delayed Predator-Prey Model With Predator Migration

Nianatus Sholihah, Ari Kusumastuti

Mathematics Department, State Islamic University of Maulana Malik Ibrahim Malang

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ABSTRACT

The predator-prey model is one of the interaction models between at least two types of species in the form of ordinary nonlinear differential equations. In this model, based on the availability of prey then we add of time delay with predator migration to describe biological control. In this paper, We will analyze the existance and stability of equilibria. The main theoritical results are perform numerical simulations on changes in the initial value of a population to determine the extent to which human control is done as a predator migration.

Corresponding Author:

Nianatus Sholihah,
Mathematics Department,
State Islamic University of Maulana Malik Ibrahim Malang
Email: nianatus12@gmail.com

1. INTRODUCTION

A fundamental type of interactions which effects population dynamics of all species is predation. The predator–prey relationship is ubiquitous in the nature and hence it has been one of the dominant themes in mathematical biology. Since the pioneering work of Lotka and Volterra, various predator–prey models have been built by incorporating additional biological processes into the classical Lotka–Volterra predator–prey equations. [1] Biological control is an approach in controlling insects and minimizing losses resulting from insect infestations.

In this section the predator-prey model that will be discussed refer to Chen (2013) that is predator-prey model with predator migration and delay time which can be written as follows:

$$\frac{du(t)}{dt} = u(t)[1 - av(t)],$$

$$\frac{dv(t)}{dt} = v(t)[-d + bu(t-\tau) - v(t)] + m[u(t) - v(t)]. \qquad (1a)$$

The focus of this discussion is to determine the stability requirements of the predator-prey model with predator migration and delay time into the table so that it can perform the simulation in accordance with the conditions already obtained. Furthermore, the simulation is to change the initial value of a population which can lead to how much the role of predator migration so that equilibrium in the population remains proportional.

The model Chen (2013) is divided into one prey and two predators, ie predator level I and predator migration. The predator populations of level I in this case are biomass predator and predator migration is the man who acts as biological control, while the prey population is biomass prey.

3. RESULTS AND DISCUSSION

Stapes in this research is the determination of stability model and simulation. In determining the stability we will find the value from its fixed point, then linearize the model to get the Jacobian matrix and stability analysis to get the equilibrium point. For simulations two things are done, simulating for baseline values when assuming more populations of predators than large prey populations and for simulating both initial values when large numbers of prey populations are more than many predator populations.

1. Existance of equilibria and their linear stability

To get the equilibrium point of this predator-prey model, we will find the fixed point first. By definition The equilibrium point of the system (1a) Obtained when

$$\frac{du(t)}{dt} = u(t)[1 - av(t)] = 0, \tag{2a}$$

$$\frac{dv(t)}{dt} = v(t)[-d + bu(t-\tau) - v(t)] + m[u(t) - v(t)] = 0. \tag{2b}$$

Of the equation then obtained three fixed point. First equilibrium point is $O = (0,0)$ then, the second equilibrium is $E_0 = \big(0, -(m+d)\big)$ Which always exists and $E^* = \left(\frac{1+a(m+d)}{a(am+b)}, \frac{1}{a}\right)$. After getting the equilibrium point then will be analyzed the stability.

Linearization is one of the methods used to transform nonlinear differential equations into linear differential equations by expanding the Taylor series and eliminating nonlinear tribes in the equilibrium point environment and to get matrix Jacobian. We have matrix Jacobian as follow:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 1 - av^* & au^* \\ bv^* + m & -d + bu^* - 2v^* - m \end{bmatrix} \tag{3a}$$

of the matrix jacobi obtained, then will be analyzed stability.

To determine the stability at the equilibrium point, we will calculate the eigenvalues from the equilibrium point. At the first equilibrium point ie $O = (0,0)$ then obtained eigen value 1 and $-(m+d)$. Then it can be said that O always *unstable*. The equilibrium $E_0 = \big(0, -(m+d)\big)$ then obtained eigen value $1 + a(m+d)$ and $m + d$. If $m + d < 0$ Then the negative real eigen value is obtained, thus E_0 always *exists*. if $1 + a(m+d) < 0$ then E_0 is *stable*. If $1 + a(m+d) > 0$ so E_0 is *unstable*. For the equilibrium $E^* = \left(\frac{1+a(m+d)}{a(am+b)}, \frac{1}{a}\right)$ then obtained eigen value $P + \frac{\sqrt{P^2+4Q}}{2}$ and $P - \frac{\sqrt{P^2+4Q}}{2}$. If value from $(\lambda_5 + \lambda_6) < 0$ and $(\lambda_5\lambda_6) < 0$ then the equilibrium E^* is *asymptotically stable*.

From the eigen value obtained is then given the stability requirements in the table 1.

Table 1. Stability condition of predator-prey equilibrium point with predator migration and time delay

Equilibrium point	Stability requirements	Explanation
$\boldsymbol{O = (0,0)}$	*unstable*	-
$\boldsymbol{E_0 = \big(0, -(m+d)\big)}$	*stable*	$1 + a(m+d) < 0$
	unstable	$1 + a(m+d) > 0$
$\boldsymbol{E^* = \left(\frac{1+a(m+d)}{a(b+ma)}, \frac{1}{a}\right)}$	*asymptotically stable*	$(\lambda_5 + \lambda_6) < 0$ and $(\lambda_5\lambda_6) < 0$

2. Simulation for initial value variation

In this section two simulations will be given from the equation (1a). First, Simulated predator-prey model with predator migration and delay time for initial population value cases when it is assumed that the population of prey is more than the predator population by using parameter value in table 2.

Table 2. Parameter values of population cases prey > predator

Simulation	$\boldsymbol{a}$	$\boldsymbol{d}$	$\boldsymbol{b}$	$\boldsymbol{\tau}$	$\boldsymbol{u(t)}$	$\boldsymbol{v(t)}$
i	4	0.5	2	0.5	0.8	0.5
					0.6	0.3
ii	1	1	2	0.5	0.8	0.5
					0.6	0.3

Through the Simulation parameter value (i), To determine the value of how much predator migration can be determined from the provisions of Proposition (1).

Proposition 1

i. If $m > d$ then (2) has two equilibria O and E^*
ii. If $-d - \frac{1}{a} < m < -d$ then (2) has three equilibria O, E_0 and E^*
iii. If $m \leq -d - \frac{1}{a}$ then (2) has two equilibria O and E_0 [1]

Thus it can be taken the value of the predator migration is equal $m = 1$. When the value of $m = 1$ Then the equilibrium obtained is E^*.

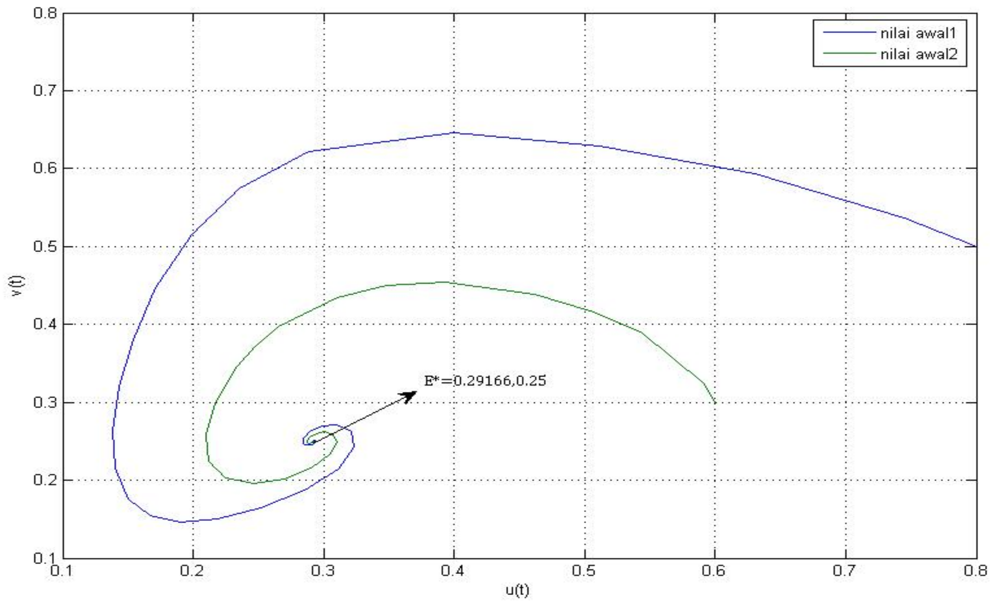

Figure 1. Numerical simulations i case the population prey > predator

Based on Simulation parameter value (ii), have two value m. When $m = 0$ then The equilibrium obtained is E^*. Then, when the value of $m = -2$ the equilibrium point obtained is E_0. But In Table 3.3 it is explained that the equilibrium condition E_0 unstable when the value of $1 + a(m + d) > 0$. So, the equilibrium condition E_0 is unstable. As a result, there are two points of interior equilibrium that exist, ie $E^* = (1,1)$ dan $E_0 = (0,1)$. But only the equilibrium point E^* Which is asymptotically stable while E_0 is unstable. the equilibrium point E^* asymtotically stable, This is because the whole solution with different initial values in the population prey > predator toward the equilibrium point E^*.

Phase photographs in Figure 1 show that the equilibrium point E^* asymtotically stable, This is because the whole solution with different initial values in the population prey > predator toward the equilibrium point E^*.

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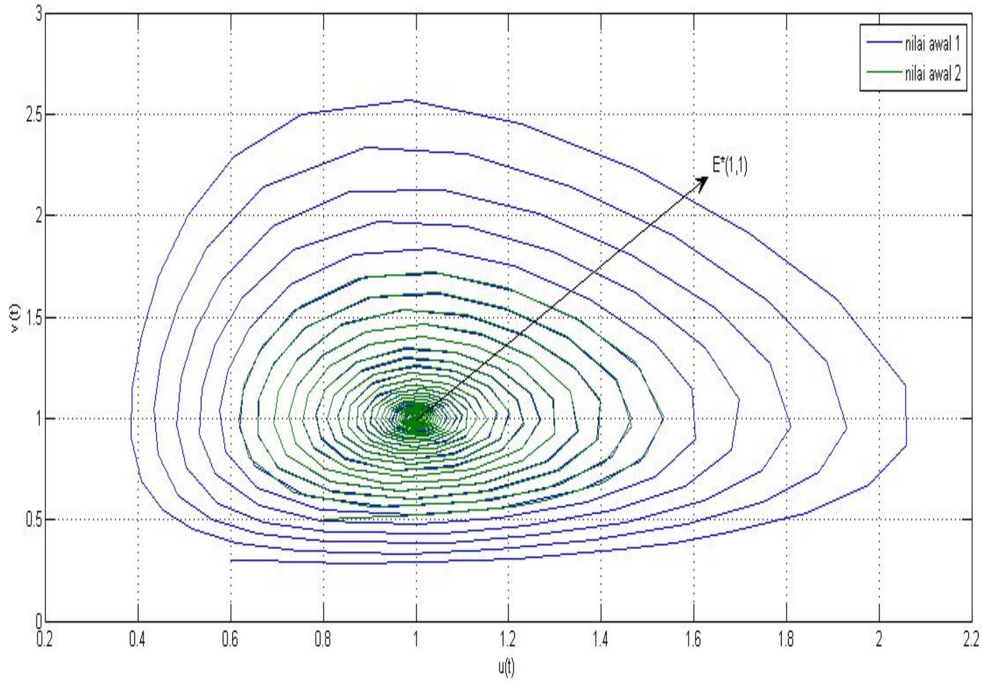

Figure 2. Numerical simulations case ii the population prey > predator

Furthermore, simulated predator-prey model with predator migration and delay time for population cases when the predator population is more than the number of prey populations using parameter values in table:

Table 3. Parameter values of population cases predator > prey

a	d	b	τ	$u(t)$	$v(t)$
2	0	2	0.5	0.6	0.8
				0.4	0.9

Through the value of these parameters, Thus can be taken of great value $m = -2$. When value of $m = -2$ Then the equilibrium obtained is the equilibrium point equilibrium E_0. In Table 3.3 it is explained that the equilibrium condition E_0 stable when the value of $1 + a(m + d) < 0$. Because the condition is met then the equilibrium point E_0 is stable. Thus, the simulation results in Figure 3. correspond to the analysis.

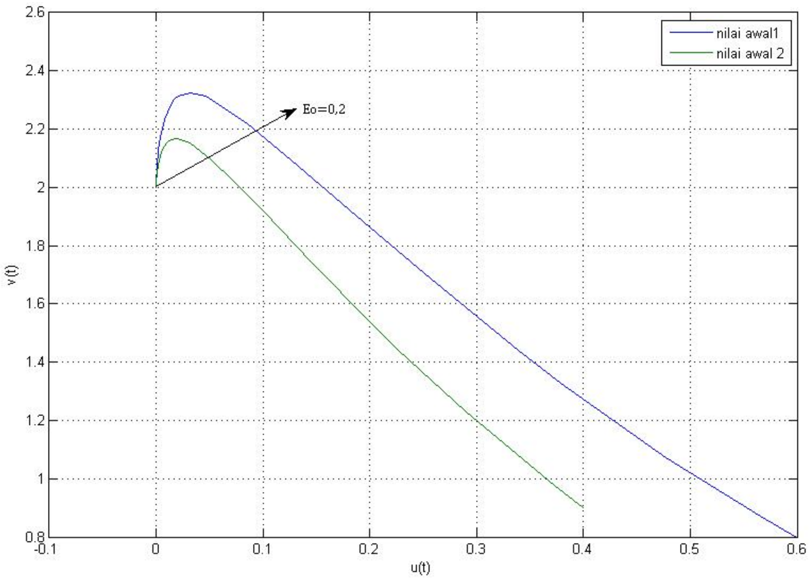

Figure 3. Numerical simulations case the population predator $>$ prey

CONCLUSION

Based on the result and discussion, The stability gained for a model in this aquatic environment is $E_0 = \left(0, -(m+d)\right)$ stable when $1 + a(m+d) < 0$ and $E^* = \left(\frac{1+a(m+d)}{a(b+ma)}, \frac{1}{a}\right)$ asymptotically stable when $(\lambda_5 + \lambda_6) < 0$ and $(\lambda_5 \lambda_6) < 0$ with predator migration is the human control in terms of harvesting it becomes its control. In the simulation of prey population cases more than the predator population is the equilibrium obtained is $E^* = \left(\frac{7}{24}, \frac{1}{4}\right)$ and $E^* = (1,1)$. For the second case where the predator population value is more than the prey population in getting equilibrium $E_0 = (0,2)$.

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REFERENCES

[1] Y. Chen, "Dynamics of a delayed predator–prey model with predator migration," *Applied Mathematical Modelling*, vol. 37, p. 1400, 2013.