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CATEGORIZATION OF STUDENT ERRORS IN SOLVING INTEGRAL PROBLEMS

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Abstract. This research aims to categorise and describe students' errors in solving integral problems. This study was conducted on students of the second semester of mathematics education majoring in 40 students. Data retrieval is obtained by giving a test. The results of student work are grouped according to errors that appear from the results of the problem-solving. The categorization of student errors in solving integral problems obtained from this study are (1) errors in the selection of problem solving strategies, (2) errors in sampling, (3) error in integration is the addition of a constant (C) to the integral of course, and (4) errors in calculation because not careful.

Keywords. Categorization; Error; Problem Solving; Integral

A. INTRODUCTION

Calculus is a compulsory subject that must be taken by students who study mathematics or mathematics education. Calculus contains material including differentials and integrals. After studying differential, students learn about integrals. So when learning about integrals, students are assumed to have mastered the differential completely.

In Calculus, most of the material was discussed again in public high schools but with more depth. Even so, there are still many students who find it difficult to study calculus. From the results of tests on 40 first year math education students, it was found that 50% scored below 60 from a range of 100.

Some studies state that many students have difficulties in studying calculus (Grehan, O'Shea, & Bhaird, 2010; Makgaka & Maknakwa, 2016; Muzangwa & Chifamba, 2012; Tarmizi, 2010; Zakaria & Salleh, 2015). Tarmizi (2010) investigate the student's performance in solving the addition function by examining the stages. Zakaria & Salleh (2015) investigate student perceptions of integral learning difficulties and their readiness for integral learning using technology.

Other researchers examine the difficulty of students in visualizing the definite integral in a diagram (Huang, 2014). One application of the integral is of course to calculate the area. So that students' skills are needed in drawing the graph and looking for the area.

Based on the description, the researcher is interested in examining the mistakes made by students when completing substitution integral questions. This study aims to categorize and describe errors that occur when solving integral substitution problems.

B. RESEARCH METHOD

This study is classified as descriptive research conducted on students of mathematics education majoring at UIN Maulana Malik Ibrahim Malang in the second semester totalling 40 students. The main instruments of the study are the researchers themselves and student worksheets. Data retrieval is obtained from the test results given to students. Student test results are grouped

according to the category of errors made. The assignments given to students are presented in Figure 1.

$$\text{Determine the results from } \int_1^4 \frac{1}{\sqrt{t}(\sqrt{t}+1)^3} dt$$

Figure 1. Tasks are given to students

One alternative solution to the questions given to students is presented in Figure 2.

Solving questions

Suppose $u = \sqrt{t} + 1$ then $\frac{du}{dt} = \frac{1}{2\sqrt{t}}$ so that $dt = 2\sqrt{t} du$.

With that

$$\begin{aligned} \int_1^4 \frac{1}{\sqrt{t}(\sqrt{t}+1)^3} dt &= \int_1^4 \frac{1}{\sqrt{t}u^3} 2\sqrt{t} du \\ &= 2 \int_1^4 \frac{1}{u^3} du \\ &= \frac{2}{-3+1} u^{-3+1} \Big|_1^4 \\ &= -u^{-2} \Big|_1^4 \\ &= -\frac{1}{(\sqrt{t}+1)^2} \Big|_1^4 \\ &= -\frac{1}{(\sqrt{4}+1)^2} + \frac{1}{(\sqrt{1}+1)^2} \\ &= -\frac{1}{9} + \frac{1}{4} = -\frac{4}{36} + \frac{9}{36} = \frac{5}{36} \end{aligned}$$

So $\int_1^4 \frac{1}{\sqrt{t}(\sqrt{t}+1)^3} dt = \frac{5}{36}$

Figure 2. Completion of Integral Problems

C. RESULT AND DISCUSSION

Based on the results of the tests given, 5 students answered correctly and 35 were wrong. Errors made by students in solving integral questions can be categorized into four categories, as presented in Table 1.

Table 1: Error Category

Error Category	Frequency
Error in selecting problem-solving strategies	11
Error in Sampling	7
Error in integration is the addition of a constant (C) to the integral of course	13
Errors in Calculations Because Not Careful	4

Errors in solving integral problems are further explained below.

1. Error in selecting problem-solving strategies

Errors made by students in solving questions in this category are wrong in choosing a settlement strategy. Without conducting a test, students immediately integrate this should not be done. Errors in choosing a settlement strategy show that students have not mastered the integral concept of substitution. Errors in choosing a settlement strategy are presented in Figure 3.

$$\begin{aligned}
 2) \int_1^4 \frac{1}{\sqrt{t}(\sqrt{t}+1)^3} dt &= \int_1^4 (\sqrt{t}(\sqrt{t}+1)^3)^{-1} \\
 &= \int_1^4 (t)^{\frac{1}{2}} (\sqrt{t}+1)^{-3} \\
 &= \int_1^4 (t)^{-\frac{1}{2}} (\sqrt{t}+1)^{-3} \rightarrow \\
 &= \left[2(t)^{\frac{1}{2}} \left(\frac{1}{2} \right) (\sqrt{t}+1)^{-2} \right]_1^4 \\
 &= \left[-1\sqrt{t} (\sqrt{t}+1)^{-2} \right]_1^4 \\
 &= \left[\frac{-1\sqrt{t}}{(\sqrt{t}+1)^2} \right]_1^4 \\
 &= \left[\frac{-1\sqrt{4}}{(\sqrt{4}+1)^2} \right] - \left[\frac{-1\sqrt{1}}{(\sqrt{1}+1)^2} \right] \\
 &= \left[-2 \cdot \frac{1}{9} \right] - \left[-1 \cdot \frac{1}{4} \right] \\
 &= -\frac{2}{9} + \frac{1}{4} \\
 &= \frac{-8+9}{36} \\
 &= \frac{1}{36}
 \end{aligned}$$

Figure 3. Error in selecting a settlement strategy

Completion of the questions can be done without a comparison, but must be explained before being integrated. Seen in Figure 3, students immediately integrate without elaboration in advance so that errors occur in the integration of the problem.

2. Error in Sampling

In this category, students already understand that the integral questions given are substitution integrals. This is indicated by the choice of strategy. Students already know the first step is to do the research. Student work is presented in Figure 4.

Handwritten student work for Figure 4:

$$\int_1^4 \frac{1}{\sqrt{t} (\sqrt{t} + 1)^3} dt$$

$$\int_1^4 \frac{1}{t^{1/2} (t^{1/2} + 1)^3}$$

$$\int_1^4 t^{-1/2} (t^{1/2} + 1)^{-3}$$

misalkan

$$u = t^{1/2} + 1$$

$$\frac{du}{dt} = \frac{1}{2} t^{-1/2} = \frac{1}{2} t^{-1/2}$$

$$\frac{2}{3} du = t^{-1/2}$$

$$= \int_1^4 u^{-3} \frac{2}{3} du$$

1 -2 1

Figure 4. Error in isolation

From Figure 4, it is known that students make an example $u = \sqrt{t} + 1$. When determination $\frac{du}{dt}$ there is an error. Should be obtained $\frac{du}{dt} = \frac{1}{2} t^{-1/2}$, determined derivatives but students integrate it so that an error occurs.

Another mistake that occurred during the analysis was in determining the derivative. Students have understood if after searching for the derivative but there is an error in the process of decline. The results of student work are presented in Figure 5.

Handwritten student work for Figure 5:

2) Integral dari $\int_1^4 \frac{1}{\sqrt{t} (\sqrt{t} + 1)^3} dt$

$$\int_1^4 \frac{1}{\sqrt{t} (\sqrt{t} + 1)^3} dt = \int_1^4 u^{-3} dt$$

misal $u = (\sqrt{t} + 1)$

$$\frac{dt}{\sqrt{t}} = 1$$

$$\frac{1}{\sqrt{t}} = dt$$

$$= \int_1^4 u^{-3} \frac{1}{\sqrt{t}} dt$$

$$= \left[-\frac{1}{2} u^{-2} \frac{1}{\sqrt{t}} \right]_1^4 + C$$

$$= -\frac{1}{2} (\sqrt{t} + 1)^{-2} \frac{1}{\sqrt{t}} + C$$

$$= -\frac{1}{2} \sqrt{t}$$

Figure 5. Error in isolation

In Figure 5 the student is correct in determining the comparison with $u = \sqrt{t} + 1$ but results from derivative that is wrong.

3. Error in integration is the addition of a constant (C) to the integral of course

The error at this stage is when integration after sampling. Students add a constant (C). The integral questions given are of course an integral. So there is no need to add a constant (C) during integration. Student work is presented in Figure 6.

Handwritten student work for Figure 6:

Diketahui :

$$\int_1^9 t^{-\frac{1}{2}} (\sqrt{t} + 1)^{-3} dt$$

Misalkan $u = \sqrt{t} + 1$
 $= t^{\frac{1}{2}} + 1$
 $du = \frac{1}{2} t^{-\frac{1}{2}} dt$

Sehingga :

$$= \int_1^9 2 \cdot du \cdot u^{-3}$$

$$= 2 \int_1^9 u^{-3} du$$

$$= 2 \left[-\frac{1}{2} u^{-2} + C \right]_1^9$$

$$= 2 \left[-\frac{1}{2} u^{-2} \right]_1^9$$

$$= \left[-1 u^{-2} \right]_1^9$$

$$= \left[-\frac{1}{u^2} \right]_1^9$$

$$= \frac{-1}{9^2} - \frac{-1}{1^2}$$

The student incorrectly adds a constant C in the integration step, which is circled in red.

Figure 6. Error in integration is the addition of a constant (C)

4. Errors in Calculations Because Not Careful

At this stage, students have been able to integrate properly. From the initial stage it is made an example, then stated in the form of $U du$. Student work is presented in Figure 7.

Handwritten student work for Figure 7:

$$\int_1^9 \frac{1}{\sqrt{t} (\sqrt{t} + 1)^3} dt$$

$$= \int_1^9 \frac{1}{t^{\frac{1}{2}} (t^{\frac{1}{2}} + 1)^3} dt$$

$$= \int_1^9 t^{-\frac{1}{2}} (t^{\frac{1}{2}} + 1)^{-3} dt \quad \checkmark$$

Untuk memudahkan penyelesaian,
 Misalkan $u = (t^{\frac{1}{2}} + 1)$
 $du = \frac{1}{2} t^{-\frac{1}{2}} dt$
 $2 du = t^{-\frac{1}{2}} dt \quad \checkmark$

Sehingga dapat ditulis

$$= \int_1^9 u^{-3} 2 du$$

$$= 2 \int_1^9 u^{-3} du$$

The student correctly identifies the substitution and the differential, and the final result is partially shown.

Figure 7. Students can make an example and change the function to form $U du$

Then students integrate, substitute the upper and lower limits. But when performing count operations for simplification errors occur. Students have understood the concept of integral substitution. Errors made can occur due to lack of scrutiny in calculations. The results of student work when making mistakes are presented in Figure 8.

$$\begin{aligned}
 &= 2 \left[\frac{1}{-2} (\sqrt{t}+1)^{-2} \right] \\
 &= 2 \left[\frac{1}{-2} (\sqrt{t}+1)^2 \right] \\
 &= 2 \left[\left(\frac{1}{-2} \frac{1}{(\sqrt{t}+1)^2} \right) - \left(\frac{1}{-2} \frac{1}{(\sqrt{t}+1)^2} \right) \right] \\
 &= 2 \left[\left(\frac{1}{-2} \right) \left(\frac{1}{(2+1)^3} - \frac{1}{(1+1)^2} \right) \right] \\
 &= (-1) \left[\frac{1}{3^3} - \frac{1}{2^2} \right] \\
 &= (-1) \left[\frac{1}{9} - \frac{1}{4} \right] \\
 &= (-1) \left[\frac{4-9}{36} \right] \\
 &= (-1) \left(\frac{-5}{36} \right) \\
 &= \frac{5}{36}
 \end{aligned}$$

Figure 8. Error in counting operations

D. CONCLUSION

Students' errors in solving integral substitution problems can be categorized into 4 categories, namely (1) errors in the selection of problem solving strategies, (2) errors in sampling, (3) error in integration is the addition of a constant (C) to the integral of course, and (4) errors in calculation because not careful. Errors that occur in categories (1) and (2) are more due to students not understanding the concept of integral substitution. Because they lack understanding of the concept, students choose the settlement strategy and experience confusion to determine the problem-solving step.

Errors in the category (3) occur because students do not understand the exact integral so that there is a constant addition (C). Errors in the category (4) occur because students are not careful in calculating. Less thorough can occur because students work limited time so that they are in a hurry to complete.

REFERENCES

- Grehan, M., O'Shea, A., & Bhaird, C. M. an. (2010). How do students deal with difficulties in mathematics? *CETL-MSOR Conference 2010*, (September 2009), 34–71. Retrieved from <http://www.mathstore.ac.uk/headocs/Proceedings2010.pdf#page=34>
- Huang, C. H. (2014). Calculus Students' Visual Thinking of Definite Integral. *American Journal of Educational Research*, 3(4), 476–482. <https://doi.org/10.12691/education-3-4-14>
- Makgaka, S., & Maknakwa, E. G. (2016). Exploring Learners' Difficulties in Solving Grade 12

- Differential Calculus : a Case Study of One Secondary School in Polokwane District. *Towards Effective Teaching and Meaningful Learning in Mathematics, Science and Technology*, 13–25.
- Muzangwa, J., & Chifamba, P. (2012). *Analysis of Errors and Misconceptions in the Learning of Calculus By Undergraduate Students*. 19(2).
- Tarmizi, R. A. (2010). Visualizing students' difficulties in learning calculus. *Procedia - Social and Behavioral Sciences*, 8, 377–383. <https://doi.org/10.1016/j.sbspro.2010.12.053>
- Zakaria, E., & Salleh, T. S. (2015). Using Technology in Learning Integral Calculus. *Mediterranean Journal of Social Sciences*, (September), 1–6. <https://doi.org/10.5901/mjss.2015.v6n5s1p144>